FTD-ID(RS)T-1218-85

AD-A164 320

FOREIGN TECHNOLOGY DIVISION

AIR FORCE SYSTEMS COMMAND

THERMAL CALCULATION OF A THERMOGENERATOR AT CHANGING TEMPERATURES ALONG THERMOCONTACT SURFACES

by

G.A. Varshavskiy, I.A. Rezgol'

FOREIGN TECHNOLOGY DIVISION

Approved for public release;
distribution unlimited.

86 2 18 041

EDITED TRANSLATION

FTD-ID(RS)T-1218-85 13 Jan 86

MICROFICHE NR: FTD-86-C-001370

THERMAL CALCULATION OF A THERMOGENERATOR AT CHANGING TEMPERATURES ALONG THERMOCONTACT SURFACES

By: G.A. Varshavskiy, I.A. Rezgol'

English pages: 14

Source: Izvestiya Akademii Nauk SSSR, Energetika i Transport, Vol. 64, Nr. 6, 1964, pp. 735-742

Country of origin: USSR
Translated by: Robert D. Hill
Requester: FTD/TQTD
Approved for public release; distribution unlimited.

PREPARED BY:

TRANSLATION DIVISION
FOREIGN TECHNOLOGY DIVISION
WP-AFB, OHIO.

U. S. BOARD ON GEOGRAPHIC NAMES TRANSLITERATION SYSTEM

Block	Italic	Transliteration	Block	Italic	Transliteration
А а	*А а*	A, a	Р р	*Р р*	R, r
Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

*ye initially, after vowels, and after ъ, ь; e elsewhere.
When written as ё in Russian, transliterate as yё or ё.

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English	Russian	English	Russian	English
sin	sin	sh	sinh	arc sh	$\sinh^{-1}$
cos	cos	ch	cosh	arc ch	$\cosh^{-1}$
tg	tan	th	tanh	arc th	$\tanh^{-1}$
ctg	cot	cth	coth	arc cth	$\coth^{-1}$
sec	sec	sch	sech	arc sch	sech^{-1}
cosec	csc	csch	csch	arc csch	csch^{-1}

Russian	English
rot	curl
lg	log

GRAPHICS DISCLAIMER

All figures, graphics, tables, equations, etc. merged into this translation were extracted from the best quality copy available.

THERMAL CALCULATION OF A THERMOGENERATOR AT CHANGING TEMPERATURES ALONG THERMOCONTACT SURFACES

G. A. Varshavskiy, I. A. Rezgol'
(Moscow)

The article finds expressions for the output power of efficiency of a thermoelectric generator and temperature distribution along the heat carrier under the condition that temperatures of the thermocontact surfaces vary due to the cooling and heating of the heat carriers. Simple approximation calculation formulas are given for the particular cases examined.

In connection with successes in semiconductor technology, there is widespread use of thermoelectric converters for obtaining electrical energy and for cooling purposes. The thermotechnical calculation and investigation of the operation of the converters are usually done by means of elementary theoretical analysis proposed by A. F. Ioffe in his time [1].and refined somewhat later by A. I. Burshteyn [2] and accepted in all the world literature [3-5].

The established methods of calculation make it possible to determine the efficiency of these devices and find the optimal conditions of their operation at constant temperatures of hot and cold soldering This simplification, being scheme plan, can be justified by two considerations of a practical nature. The first reflects the technical side of the question, since models of thermoelectric generators (TEG), created at the initial stage of development of this branch of technology, functioned at practically constant temperatures of hot and cold junctions. These are, for example, the thermoelectric generator on a kerosene lamp [1-3], the thermogenerator of apparatus SNAP-III with an isotopic heat source [6], different solar thermogenerators [7, 8], and so on. The second circumstance is connected with the fact that thermoelectric parameters of the substance greatly

depend on temperature, and a detailed account of this dependence for variable temperature limits leads to expression which cannot be investigated analytically to the end.

The striving to increase the power of the power installations inevitably led to the creation of a two-circuit scheme of the thermogenerator [9], where the transporting of the heat is produced by heat carriers, the temperature of which considerably varies as a result of the heat transfer from a hot liquid to a cold one along the channel of the heat carrier. With the thorough heat-engineering calculation of such apparatuses, the dependence of thermoelectric characteristics of the materials on temperature must be considered.

However, in the preliminary analysis of the operation of the thermogenerator, it is expedient to determine its output parameters without taking into account the effect of variable temperatures of the solderings on thermoelectric properties of the substances but considering the change in temperatures of heat carriers along the thermocontact surfaces.

This work is devoted to the finding of analytical bonds, which make it possible to make the calculation of the thermogenerator at changing (as a result of heat transfer) temperatures of the solderings. The obtained expressions can be useful in the preliminary determination of the optimal parameters of the generators (maximal power and efficiency, minimal weight, etc.) and an analysis of processes of regulation.

Given at the end of the article are certain recommendations according to an analysis of the TEG operation, taking into account the dependence of thermoelectric properties of materials on temperature.

Calculation of characteristics of the thermoelectric generator. Let us examine the TEG operation in the two-circuit scheme (Fig. 1). The hot heat carrier 1, which circulates in the first circuit, feeds heat through the "thermocontact packet" 2 to the hot junctions [solderings] of the thermoelements. Part of this heat is converted into electrical energy, and the remaining heat is drawn off from the cold junctions of the thermoelements through the "thermocontact packet" 4 to the cold heat carrier 5 circulating in the second circuit. The

thermogenerator includes a great number of single thermoelements, which are connected electrically to each other in series or in parallel, depending on the required output parameters of the TEG. The basic dimensions of the thermoelements, height h and cross sections S_p and S_n of the branches, are identical for the whole thermogenerator.

In the course of the analysis we will use the average values of the characteristics for the positive and negative branches of the thermoelement (symbols are given at the end of the article)

$$\alpha = \frac{1}{2}(\alpha_p + \alpha_n), \quad \varkappa = \varkappa_p \frac{S_p}{S_0} + \varkappa_n \frac{S_n}{S_0},$$

$$\rho = \frac{1}{4}\left(\rho_p \frac{S_0}{S_p} + \rho_n \frac{S_0}{S_n}\right), \quad j = j_p \frac{S_p}{S_0} + j_n \frac{S_n}{S_0}, \quad z = \alpha^2 / \varkappa\rho.$$

Let us take the following simplifying assumptions:

1) the electrophysical characteristics α, $\varkappa$, ρ, averaged over the operating temperature range, are identical for all the thermoelements;

2) the average current density j is constant for any cross section of the thermogenerator;

3) the thermal resistances k_r and k_x of the "thermocontact packets" are identical for all the thermoelements, and the corresponding temperature drops are proportional to the thermal flows.

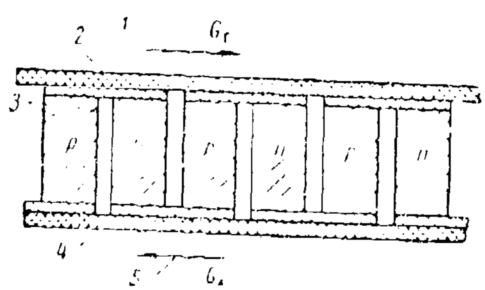

Fig. 1.

As usual [1-3], we do not consider the effect of the Thomson effect (since the specific thermal emf α is averaged over the operating temperature range), and we assume that the Joule heat liberated in the thermoelement is transported equally to the hot and cold junctions.

In this case, for the "hot" and "cold" thermocontact surfaces of the generator selected along the channel of heat carriers of elements dF (Fig. 2), we obtain correspondingly the following relations:

$$dQ_r = -G_r c_r dT_r, \qquad -G_r c_r dT_r = \frac{(T_r - t_r)}{k_r} dF$$

$$\frac{T_r - t_r}{k_r} = \frac{\varkappa}{h}(t_r - t_x) + j\alpha t_r - \frac{1}{2} j^2 \rho h$$

$$dQ_x = \pm G_x c_x dT_x, \tag{1}$$

$$\pm G_x c_x dT_x = \frac{(t_x - T_x)}{k_x} dF$$

$$\frac{t_x - T_x}{k_x} = \frac{\varkappa}{h}(t_r - t_x) + j\alpha t_x + \frac{1}{2} j^2 \rho h.$$

The solution to these equations makes it possible to find the temperature distribution along the thermocontact surfaces of the generator, the fed and removed quantities of heat, electrical power, efficiency, and so on.

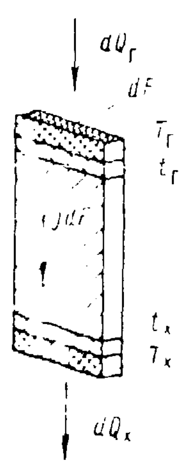

Fig. 2.

Here and further, when there is a dual sign ($\pm$ or $\mp$), the upper sign refers to the case of cocurrent flow and the lower sign, to the case of countercurrent flow.

Let us introduce the dimensionless parameters

$$\beta = \frac{\rho u h}{\varkappa}; \quad \lambda = \frac{1}{2}\left(\frac{1}{G_{\mathrm{T}} c_{\mathrm{T}}} + \frac{1}{G_{\mathrm{x}} c_{\mathrm{x}}}\right)\frac{\varkappa \Gamma_{\mathrm{T}}}{h}, \quad \mu = \frac{1}{2}\left(k_{\mathrm{T}} + k_{\mathrm{x}}\right)\frac{\varkappa}{h}$$

$$a = \frac{G_{\mathrm{T}} c_{\mathrm{T}} - G_{\mathrm{x}} c_{\mathrm{x}}}{G_{\mathrm{T}} c_{\mathrm{T}} + G_{\mathrm{x}} c_{\mathrm{x}}}, \quad b = \frac{k_{\mathrm{T}} - k_{\mathrm{x}}}{k_{\mathrm{T}} + k_{\mathrm{x}}}. \tag{2}$$

Equations (1) in this case take the following form:

$$\begin{aligned} &-\frac{1}{(1-a)\lambda}\frac{dx}{dm} = \frac{x - x'}{(1+b)\mu}, \qquad \frac{x - x'}{(1+b)\mu} = (1+\beta)x' - y' - \frac{1}{2}\beta^2 \\ &\pm\frac{1}{(1+a)\lambda}\frac{dy}{dm} = \frac{y' - y}{(1-b)\mu}, \qquad \frac{y' - y}{(1-b)\mu} = x' - (1-\beta)y' + \frac{1}{2}\beta^2. \end{aligned} \tag{3}$$

Eliminating x' and y', we obtain a system of two differential equations of the first order

$$\begin{aligned} &\frac{[1 + (1+\beta)(1+b)\mu]}{(1-a)\lambda}\frac{dx}{dm} \mp \frac{(1-b)\mu}{(1+a)\lambda}\frac{dy}{dm} + (1+\beta)x - y - \frac{1}{2}\beta^2 = 0 \\ &\frac{(1+b)\mu}{(1-a)\lambda}\frac{dx}{dm} \mp \frac{[1 + (1-\beta)(1-b)\mu]}{(1+a)\lambda}\frac{dy}{dm} + x - (1-\beta)y + \frac{1}{2}\beta^2 = 0. \end{aligned} \tag{4}$$

The solution to this equation for the case of the cocurrent flow has the form

$$\begin{aligned} x + 1 - \frac{1}{2}\beta = \Bigg\{ &A^* \left[\mathrm{ch}\frac{\lambda\psi_1 m}{\omega} - \frac{(\varphi_1 - \varphi_2)}{2\psi_1}\mathrm{sh}\frac{\lambda\psi_1 m}{\omega}\right] + \\ &+ B^*\frac{(1-a)}{\psi_1}\mathrm{sh}\frac{\lambda\psi_1 m}{\omega}\Bigg\}\exp\left[-\frac{\lambda(\varphi_1 + \varphi_2)m}{\omega}\right], \end{aligned} \tag{5}$$

$$\begin{aligned} y + 1 + \frac{1}{2}\beta = \Bigg\{ &A^*\frac{(1-a)}{\psi_1}\mathrm{sh}\frac{\lambda\psi_1 m}{\omega} + \\ &+ B^*\left[\mathrm{ch}\frac{\lambda\psi_1 m}{\omega} + \frac{(\varphi_1 - \varphi_2)}{2\psi_1}\mathrm{sh}\frac{\lambda\psi_1 m}{\omega}\right]\Bigg\}\exp\left[-\frac{\lambda(\varphi_1 + \varphi_2)m}{\omega}\right]. \end{aligned}$$

where

$$\varphi_1 = (1-a)[1-\beta^2\mu+\beta(1+b\beta\mu)], \qquad \varphi_2 = (1+a)[1-\beta^2\mu-\beta(1+b\beta\mu)], \tag{6}$$

$$\omega = 1+2(1+b\beta)\mu-(1-b^2)\beta^2\mu^2, \qquad \psi_1 = \sqrt{\left(\frac{\varphi_1-\varphi_2}{2}\right)^2+1-a}.$$

In these expressions quantities A* and B* are integration constants, which are determined from boundary conditions.

In the design of thermogenerators, from our point of view, as the boundary conditions it is most rational to accept the determining values of the maximal and mininimal temperatures in the apparatus. Thus when m=0 we will have $x_{макс} = zT_{г}^{вх}$ and $y_{мин} = zT^{вх}$. Hence,

$$A^* = x_{макс} + 1 - \frac{1}{2}\beta = H_x, \qquad B^* = y_{мин} + 1 + \frac{1}{2}\beta = H_y. \tag{7}$$

The heat $Q_г$ fed to the generator and the heat Q_X removed from the generator, equal, respectively, to

$$Q_г = G_г c_г (T_г^{вх} - T_г^{вых}), \qquad Q_х = G_х c_х (T_х^{вых} - T_х^{вх}),$$

are determined by means of expressions

$$\frac{zh}{\varkappa F_т} Q_г^* = \frac{x_{макс} - x_{мин}}{(1-a)\lambda} = H_x\theta_1^* - H_y\theta_3^*$$

$$\frac{zh}{\varkappa F_т} Q_х^* = \frac{y_{макс} - y_{мин}}{(1+a)\lambda} = H_x\theta_3^* - H_y\theta_2^*. \tag{8}$$

We find the useful electrical power of the generator $P = Q_г - Q_X$, efficiency of the generator $\eta = (Q_г - Q_х)/Q_г$ and average value of the output voltage U, referred to a single thermoelement, in the form

$$\frac{zh}{\varkappa F_т} P^* = H_x(\theta_1^* - \theta_3^*) - H_y(\theta_3^* - \theta_2^*),$$

$$\eta^* = \frac{H_x(\theta_1^* - \theta_3^*) - H_y(\theta_3^* - \theta_2^*)}{H_x\theta_1^* - H_y\theta_3^*}$$

$$\frac{zU^*}{\alpha} = \frac{H_x(\theta_1^* - \theta_3^*) - H_y(\theta_3^* - \theta_2^*)}{\beta}, \tag{9}$$

where

$$\theta_1^* = \frac{1}{(1-a)\lambda}\left\{1-\left[\mathrm{ch}\frac{\lambda\psi_1}{\omega} - \frac{(\varphi_1-\varphi_2)}{2\psi_1}\mathrm{sh}\frac{\lambda\psi_1}{\omega}\right]\exp\left[-\frac{\lambda(\varphi_1+\varphi_2)}{\omega}\right]\right\} \quad (10)$$

$$\theta_2^* = \frac{1}{(1+a)\lambda}\left\{1-\left[\mathrm{ch}\frac{\lambda\psi_1}{\omega} + \frac{(\varphi_1-\varphi_2)}{2\psi_1}\mathrm{sh}\frac{\lambda\psi_1}{\omega}\right]\exp\left[-\frac{\lambda(\varphi_1+\varphi_2)}{\omega}\right]\right\}$$

$$\theta_3^* = \frac{1}{\lambda\psi_1}\mathrm{sh}\frac{\lambda\psi_1}{\omega}\exp\left[-\frac{\lambda(\varphi_1+\varphi_2)}{\omega}\right].$$

In solving the system of equations (4) for the case of the countercurrent flow, we get

$$x+1-\frac{1}{2}\beta = \left\{A\left[\mathrm{ch}\frac{\lambda\psi_2 m}{\omega} - \frac{(\varphi_1+\varphi_2)}{2\psi_2}\mathrm{sh}\frac{\lambda\psi_2 m}{\omega}\right] + \right.$$
$$\left. + B\frac{(1-a)}{\psi_2}\mathrm{sh}\frac{\lambda\psi_2 m}{\omega}\right\}\exp\left[-\frac{\lambda(\varphi_1-\varphi_2)m}{\omega}\right] \quad (11)$$

$$y+1+\frac{1}{2}\beta = \left\{-A\frac{(1+a)}{\psi_2}\mathrm{sh}\frac{\lambda\psi_2 m}{\omega} + \right.$$
$$\left. + B\left[\mathrm{ch}\frac{\lambda\psi_2 m}{\omega} + \frac{(\varphi_1+\varphi_2)}{2\psi_2}\mathrm{sh}\frac{\lambda\psi_2 m}{\omega}\right]\right\}\exp\left[-\frac{\lambda(\varphi_1-\varphi_2)m}{\omega}\right],$$

where

$$\psi_2 = \sqrt{\left(\frac{\varphi_1+\varphi_2}{2}\right)^2 - (1-a^2)}. \quad (6a)$$

We determine the integration constants from boundary conditions, i.e., according to the maximal and minimal temperatures in the apparatus.

This leads to $x_{\text{макс}} = zT_1^{\text{вх}}$ when $m=0$ and $y_{\text{мин}} = zT_2^{\text{вх}}$ when $m=1$.

Quantities A and B have the following values:

$$A = x_{\text{макс}} + 1 - \frac{1}{2}\beta = H_x$$

$$B = \frac{1}{\mathrm{ch}\frac{\lambda\psi_2}{\omega} + \frac{(\varphi_1+\varphi_2)}{2\psi_2}\mathrm{sh}\frac{\lambda\psi_2}{\omega}}\left[H_x\frac{(1+a)}{\psi_2}\mathrm{sh}\frac{\lambda\psi_2}{\omega} + H_y\exp\frac{\lambda(\varphi_1-\varphi_2)}{\omega}\right]. \quad (12)$$

Substituting (12) into (11), and using relations (6) and (6a), we get

$$x+1-\frac{1}{2}\beta=\frac{1}{\operatorname{ch}\frac{\lambda\psi_2}{\omega}+\frac{(\varphi_1+\varphi_2)}{2\psi_2}\operatorname{sh}\frac{\lambda\psi_2}{\omega}}\left\{H_x\left[\operatorname{ch}\frac{\lambda\psi_2(1-m)}{\omega}+\right.\right.$$

$$\left.+\frac{(\varphi_1+\varphi_2)}{2\psi_2}\operatorname{sh}\frac{\lambda\psi_2(1-m)}{\omega}\right]\exp\left[-\frac{\lambda(\varphi_1-\varphi_2)m}{\omega}\right]+$$

$$\left.+H_y\frac{(1-a)}{\psi_2}\operatorname{sh}\frac{\lambda\psi_2 m}{\omega}\cdot\exp\frac{\lambda(\varphi_1-\varphi_2)(1-m)}{\omega}\right\} \tag{13}$$

$$y+1+\frac{1}{2}\beta=\frac{1}{\operatorname{ch}\frac{\lambda\psi_2}{\omega}+\frac{(\varphi_1+\varphi_2)}{2\psi_2}\operatorname{sh}\frac{\lambda\psi_2}{\omega}}\left\{H_x\frac{(1+a)}{\psi_2}\operatorname{sh}\frac{\lambda\psi_2(1-m)}{\omega}\times\right.$$

$$\times\exp\left[-\frac{\lambda(\varphi_1-\varphi_2)m}{\omega}\right]+H_y\left[\operatorname{ch}\frac{\lambda\psi_2 m}{\omega}+\frac{(\varphi_1+\varphi_2)}{2\psi_2}\operatorname{sh}\frac{\lambda\psi_2 m}{\omega}\right]\times$$

$$\left.\times\exp\frac{\lambda(\varphi_1-\varphi_2)(1-m)}{\omega}\right\}.$$

To determine the fed and removed quantities of heat Q and Q_x, electrical power P, efficiency and average value of the output voltage U, referred to a single thermoelement, we obtain the following expressions:

$$\frac{zh}{\varkappa F_т}Q_г=\frac{x_{макс}-x_{мин}}{(1-a)\lambda}H_x\theta_1-H_y\theta_3,$$

$$\frac{zh}{\varkappa F_т}Q_x=\frac{y_{макс}-y_{мин}}{(1+a)\lambda}H_x\theta_3-H_y\theta_2$$

$$\frac{zh}{\varkappa F_т}P=H_x(\theta_1-\theta_3)-H_y(\theta_3-\theta_2), \tag{14}$$

$$\eta=\frac{H_x(\theta_1-\theta_3)-H_y(\theta_3-\theta_2)}{H_x\theta_1-H_y\theta_3}$$

$$\frac{zU}{\alpha}=\frac{H_x(\theta_1-\theta_3)-H_y(\theta_3-\theta_2)}{\beta}.$$

Here

$$\theta_1 = \frac{1}{(1-a)\lambda}\left\{1 - \frac{\exp\left[-\dfrac{\lambda(\varphi_1-\varphi_2)}{\omega}\right]}{\operatorname{ch}\dfrac{\lambda\psi_2}{\omega} + \dfrac{(\varphi_1+\varphi_2)}{2\psi_2}\operatorname{sh}\dfrac{\lambda\psi_2}{\omega}}\right\}$$

$$\theta_2 = \frac{1}{(1+a)\lambda}\left\{1 - \frac{\exp\dfrac{\lambda(\varphi_1-\varphi_2)}{\omega}}{\operatorname{ch}\dfrac{\lambda\psi_2}{\omega} + \dfrac{(\varphi_1+\varphi_2)}{2\psi_2}\operatorname{sh}\dfrac{\lambda\psi_2}{\omega}}\right\} \qquad (15)$$

$$\theta_3 = \frac{\dfrac{1}{\lambda\psi_2}\operatorname{sh}\dfrac{\lambda\psi_2}{\omega}}{\operatorname{ch}\dfrac{\lambda\psi_2}{\omega} + \dfrac{(\varphi_1+\varphi_2)}{2\psi_2}\operatorname{sh}\dfrac{\lambda\psi_2}{\omega}}.$$

It is easy to show that the limiting transition to the case of constant temperatures on each of the two thermocontact surfaces ($\lambda=0$) and the absence of thermal resistance of the "thermocontact packets" ($\mu=0$) leads to formulas unknown from literature [1-3] for output parameters of TEG.

Comparison of efficiency of the cocurrent flow and countercurrent flow. Let us make a comparison with identical $x_{макс}$ and $y_{мин}$ and identical values of β (i.e., at identical temperatures of the heat carriers at the input to the TEG and identical current densities).

Let us expand the expression for quantities $\theta_1^*, \theta_2^*, \theta_3^*, \theta_1, \theta_2, \theta_3$ in series according to powers of the parameter

$$\theta_1^* = \frac{1}{(1+a)}\left\{\frac{\varphi_1}{\omega} - \frac{\lambda[\varphi_1^2 + (1-a^2)]}{2\omega^2} + \frac{\lambda^2[\varphi_1^3 + 2(1-a^2)\varphi_1 + (1-a^2)\varphi_2]}{6\omega^3} - \dots\right\}$$

$$\theta_2^* = \frac{1}{(1+a)}\left\{\frac{\varphi_2}{\omega} - \frac{\lambda[\varphi_2^2 + (1-a^2)]}{2\omega^2} + \frac{\lambda^2[\varphi_2^3 + 2(1-a^2)\varphi_2 + (1-a^2)\varphi_1]}{6\omega^3} - \dots\right\}$$

$$\theta_3^* = \frac{1}{\omega} - \frac{\lambda(\varphi_1+\varphi_2)}{2\omega^2} + \frac{\lambda^2[\varphi_1^2 + \varphi_2^2 + \varphi_1\varphi_2 + (1-a^2)]}{6\omega^3} - \dots \qquad (16)$$

$$\theta_1 = \frac{1}{(1-a)}\left\{\frac{\varphi_2}{\omega} - \frac{\lambda[\varphi_2^2 + (1-a^2)]}{2\omega^2} + \frac{\lambda^2[\varphi_2^3 + 2(1-a^2)\varphi_2 + (1-a^2)\varphi_1]}{6\omega^3} - \dots\right\}$$

$$\theta_2 = -\frac{1}{(1+a)}\left\{\frac{\varphi_2}{\omega} - \frac{\lambda[\varphi_2^2 + (1-a^2)]}{2\omega^2} + \frac{\lambda^2[\varphi_2^3 + 4(1-a^2)\varphi_2 + (1-a^2)\varphi_1]}{6\omega^3} - \ldots\right\}$$

$$\theta_3 = \frac{1}{\omega} - \frac{\lambda(\varphi_1+\varphi_2)}{2\omega^2} + \frac{\lambda^2[\varphi_1^2 + \varphi_2^2 + 2\varphi_1\varphi_2 + 2(1-a^2)]}{6\omega^3} - \ldots$$

Using expressions (16) for determining the differences in power and efficiency (with countercurrent flow and cocurrent flow), we get

$$\frac{zh(P-P^*)}{\varkappa F_1} = H_x[(\theta_1-\theta_1^*)-(\theta_3-\theta_3^*)] - H_y[(\theta_3-\theta_3^*)-(\theta_2-\theta_2^*)] \approx$$

$$\approx \frac{\lambda^2(1-a^2)}{6\omega^3}\{(H_x-H_y)\cdot 2\beta(1+b\beta\mu) + (H_x+H_y)\beta^2[(1+b\beta\mu)^2-\beta^2\mu^2]\} > 0$$

$$\left(\frac{z}{\varkappa F}\right)^2(\eta-\eta^*) = H_x^2(\theta_1\theta_3^* - \theta_3\theta_1^*) + H_xH_y(\theta_2\theta_1^* - \theta_1\theta_2^*) + \qquad (17)$$

$$+ H_y^2(\theta_3\theta_2^* - \theta_2\theta_3^*) \approx$$

$$\approx \frac{\lambda^2(1-a^2)\beta^2}{6\omega^3}[(H_x^2-H_y^2)(1-\beta^2\mu) + (H_x^2+H_y^2)\beta(1+b\beta\mu)] > 0.$$

Thus the mode of the countercurrent flow provides higher values of the output power and efficiency of the TEG than does the mode of the cocurrent flow.

Conditions of an open circuit. When $\beta=0$ we have

$$\theta_1^* = \theta_2^* = \theta_3^* = \theta^*(0), \quad \theta_1 = \theta_2 = \theta_3 = \theta(0).$$

The average emf of the single thermoelement is determined from expressions

$$\frac{z\bar{e}^*}{a} = (x_{\text{макс}} - y_{\text{мин}})\theta^*(0) = \frac{(x_{\text{макс}} - y_{\text{мин}})\left[1 - \exp\left(-\frac{2\lambda}{1+2\mu}\right)\right]}{2\lambda},$$

$$\frac{z\bar{e}}{a} = (x_{\text{макс}} - y_{\text{мин}})\theta(0) = \frac{x_{\text{макс}} - y_{\text{мин}}}{\lambda\left[1 + a\,\mathrm{cth}\left(\frac{\lambda a}{1+2\mu}\right)\right]}. \qquad (18)$$

For the case of a=0, we get

$$\frac{\bar{z\varepsilon}}{\alpha} = \frac{x_{\text{макс}} - y_{\text{мин}}}{1+\lambda+2\mu}. \qquad (19)$$

Example of calculation of TEG parameters. For the simplest case $G_r c_r = G_X c_X$ and $k_r = k_X$ (i.e., a=0 and b=0), the calculation of output parameters of the TEG when $x_{\text{макс}} = 1$, $y_{\text{мин}} = 0.5$, $\lambda = 0$-0.5 was conducted; $\mu = 0$ and $\mu = 0.1$. Results of the calculation are given on Figs. 3 and 4. It is found that the volt-ampere characteristic of the TEG $\bar{z}U/\alpha = f_1(\beta)$ is close to a straight line, and the characteristic of the power $zhP/\varkappa F_{\text{т}} = f_2(\beta)$ is close to a quadratic parabola. Analysis of these relations shows that the output parameters of the TEG (for the case of $G_r c_r = G_X c_X$ and $k_r = k_X$) can be approximated by simple relations having the same form as those known in literature [1-3]

$$U \approx \alpha_{\text{пр}}(T_{\text{г}}^{\text{вх}} - T_{\text{х}}^{\text{вх}}) - j\rho_{\text{пр}}h, \qquad P \approx jF_{\text{т}}\alpha_{\text{пр}}(T_{\text{г}}^{\text{вх}} - T_{\text{х}}^{\text{вх}}) - j^2\rho_{\text{пр}}hF_{\text{т}} \qquad (20)$$

$$Q_{\text{г}} \approx \frac{\varkappa_{\text{пр}}}{h}F_{\text{т}}(T_{\text{г}}^{\text{вх}} - T_{\text{х}}^{\text{вх}}) + jF_{\text{т}}\alpha_{\text{пр}}T_{\text{г}}^{\text{вх}} - \frac{1}{2}j^2\rho_{\text{пр}}hF_{\text{т}}$$

$$Q_{\text{х}} \approx \frac{\varkappa_{\text{пр}}}{h}F_{\text{т}}(T_{\text{г}}^{\text{вх}} - T_{\text{х}}^{\text{вх}}) + jF_{\text{т}}\alpha_{\text{пр}}T_{\text{х}}^{\text{вх}} + \frac{1}{2}j^2\rho_{\text{пр}}hF_{\text{т}},$$

where

$$\alpha_{\text{пр}} = \frac{\alpha}{1+\lambda+2\mu}, \quad \varkappa_{\text{пр}} = \frac{\varkappa}{1+\lambda+2\mu}$$

$$\rho_{\text{пр}} = \frac{\rho\left\{1+(\lambda+2\mu)\left[1+\frac{z(T_{\text{г}}^{\text{вх}}+T_{\text{х}}^{\text{вх}})}{2}\right]\right\}}{1+\lambda+2\mu} \qquad (21)$$

$$z_{\text{пр}} = \frac{z}{1+(\lambda+2\mu)\left[1+\frac{z(T_{\text{г}}^{\text{вх}}+T_{\text{х}}^{\text{вх}})}{2}\right]}.$$

e mode of maximal power of the TEG here corresponds to values

$$U_{\text{опт}} \approx \frac{1}{2}\alpha_{\text{пр}}(T_{\text{г}}^{\text{вх}} - T_{\text{х}}^{\text{вх}}), \quad j_{\text{опт}} \approx \frac{1}{2}\frac{\alpha_{\text{пр}}(T_{\text{г}}^{\text{вх}} - T_{\text{х}}^{\text{вх}})}{\rho_{\text{пр}}h}, \qquad (22)$$

and for the mode of maximal efficiency

$$U_{\text{опт}} \approx \frac{\alpha_{\text{пр}}(T_{\text{г}}^{\text{вх}} - T_{\text{х}}^{\text{вх}})M_0}{M_0+1}, \quad j_{\text{опт}} \approx \frac{\alpha_{\text{пр}}(T_{\text{г}}^{\text{вх}} - T_{\text{х}}^{\text{вх}})}{(M_0+1)\rho_{\text{пр}}h}, \qquad (23)$$

$$M_0 = \sqrt{1+\frac{z_{\text{пр}}(T_{\text{г}}^{\text{вх}}+T_{\text{х}}^{\text{вх}})}{2}}.$$

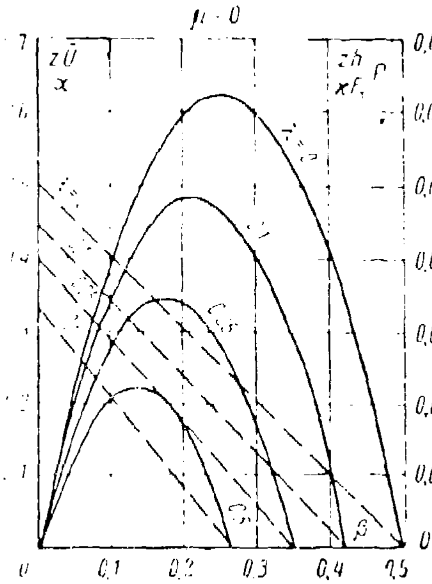

Fig. 3.

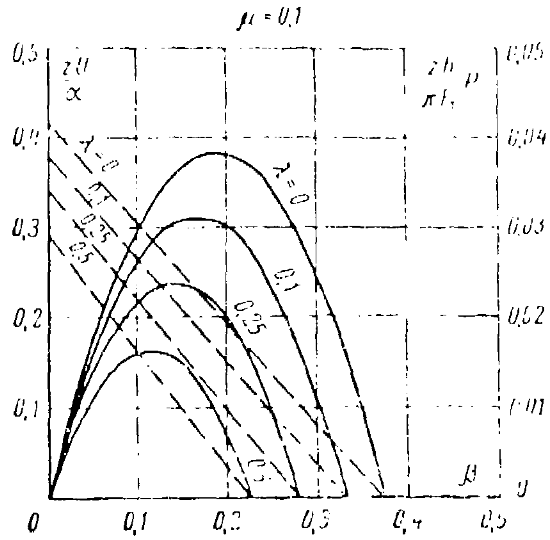

Fig. 4.

It should be noted that the expressions obtained for calculation of the basic parameters of the TEG can be used for solving other problems, for example, for an analysis of operation of the TEG with the earlier known efficiency of the discharge of heat in the second circuit and so on.

Certain recommendations on the analysis of operation of the TEG by taking into account the dependence of thermoelectric properties of materials on temperature. The use of results of the analysis given above with calculation of a specific design of the TEG gives the accuracy accepted for engineering solutions in the case of a relatively small temperature change of thermocontact surfaces along the channel of heat carriers. If these temperatures vary greatly, then the absence of the calculation of a temperature dependence of the thermoelectric properties of the materials can lead to considerable errors in the calculation. We can avoid large errors by dividing the thermogenerator into several sections with a relatively small (of the order of 10^{0}) temperature change of the thermocontact surfaces and performing a calculation for each section separately. Values of temperatures of the hot and cold heat carriers at the beginning of the first section (when m=0) can be used as the boundary conditions. In the course of the calculation, the heat being fed and removed, the

electrical power on the section being examined, and temperatures of heat carriers at the end of the section are determined. These temperatures serve as boundary conditions for calculation of the second section of the TEG with values of properties of the materials corrected for the new temperature mode. The calculation of the third section is similarly conducted, and so on.

Let us note that for calculation of the counterflow design of the TEG over the sections, it is necessary to introduce a change into the value of the integration constants appearing in the equations (11). In this case these constants are determined from boundary conditions in the same cross section of the TEG (when m=0), i.e., according to maximal values of temperatures of the hot and cold heat carriers. Here, instead of (12) we have

$$A = x_{\text{макс}} + 1 - \frac{1}{2}\beta = H_x, \qquad B = y_{\text{макс}} + 1 + \frac{1}{2}\beta. \qquad (12)$$

Corresponding changes are introduced into the calculation formulas (14). For calculation of the cocurrent flow scheme of the TEG over the sections, all the analytical expressions obtained earlier remain in force.

Symbols: T - temperature of heat carrier, $^{\circ}$K; t - temperature of junction of thermoelement, $^{\circ}$K; α - specific thermal emf of thermoelement, V/deg, $\varkappa$ - specific heat of thermoelement, W/m·deg; ρ - specific electrical resistance of thermoelelment, ohms·m: z - parameter of figure of merit of thermoelement, 1/deg; k - specific thermal resistance of "thermocontact packet" between the heat carrier and junction of the thermoelement, m^2·deg/W; G - mass flow rate of the heat carrier, kg/s; c - specific heat of heat carrier, J/kg·deg; h - height of thermoelement, m; S_p and S_n - areas of cross section of branches of thermoelement, m^2; F - thermocontact area, m^2; F_r - total thermocontact area of TEG, m^2; j - electrical current density, A/m^2; $\bar{U}$ - average output voltage referred to a single thermoelement, V; $\bar{\varepsilon}$ - average emf of single thermoelement, V; P - output electrical power of TEG, W; Q_r - thermal power consumed by the thermogenerator, W; Q_x - thermal power removed from thermogenerator, W; η - efficiency of the thermogenerator.

Subscripts: r - hot; x - cold; p - positive branch; n - negative

branch.

Superscripts: вх - input of heat carrier; вых - output of heat carrier. The output parameters of the TEG for the case of cocurrent flow are denoted by an asterisk.

Received
12 June 1964

References

1. Иоффе А. Ф. Полупроводниковые термоэлементы. Изд-во АН СССР, 1960.
2. Бурштейн А. И. Физические основы расчета полупроводниковых термоэлектрических устройств. Физматгиз, 1962.
3. Голдсмид Г. Применения термоэлектричества. Физматгиз, 1963.
4. Хатсопулос Г. Н., Кинэн Дж. Г. Термодинамика термоэлектрических генераторов. Сб. «Прямое преобразование тепловой энергии в электрическую». Госатомиздат, 1961.
5. Гарднер Э., Войзард Э. Термоэлектрические материалы охлаждающих устройств, применяющихся в космическом пространстве. Сб. «Преобразование тепла и химической энергии в электроэнергию в ракетных системах». Изд. иностр. лит., 1963.
6. Болмайер Е. В. Обсуждение элементарной конструкции термоэлектрического генератора. Сб. «Прямое преобразование тепловой энергии в электрическую». Госатомиздат, 1961.
7. Алатырцев Г. А., Баум В. А., Малевский Ю. Н., Охотин А. С. Солнечный термогенератор. Теплоэнергетика, 1961, № 3.
8. Баум В. А. Солнечный термогенератор. Теплоэнергетика, 1960, № 2.
9. Корлисс У. Ракетные двигатели для космических полетов. Изд. иностр. лит., 1962.